

Opener

The average value of $f(x) = x^2\sqrt{x^3+1}$ on the closed interval $[0, 2]$ is

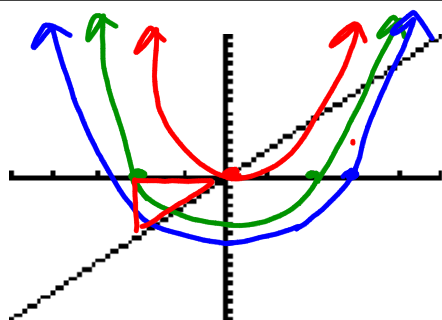
(A) $\frac{26}{9}$

(B) $\frac{13}{3}$

(C) $\frac{26}{3}$

(D) 13

(E) 26



$$\frac{1}{2}(2)(6)$$

$$A(x) = \int_0^x 3t \, dt$$

$$A(x) = \int_{-2}^x 3t \, dt$$

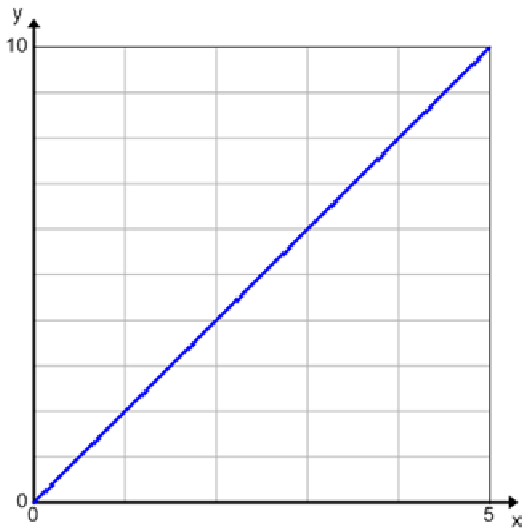
$$A(x) = \int_3^x 3t \, dt$$

5-4 day 2 The FUNdamental Theorem of Calculus

Learning Objectives:

I understand the connection between integral and differential calculus.

I can evaluate an integral using the Fundamental Theorem of Calculus Part 2.



$$g(x) = \int_0^x f(t) dt$$

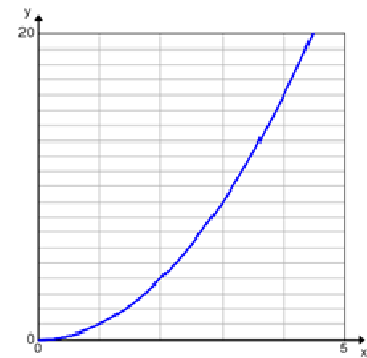
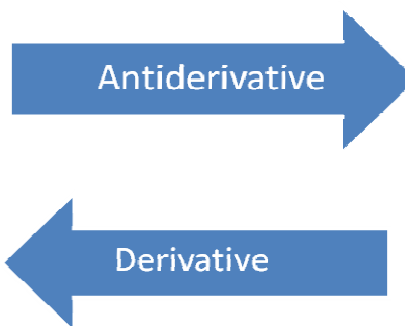
x	g(x)
0	0
1	1
2	4
3	9
4	16
5	25

$f(x) = 2x$
 $g(x) = x^2$

der \rightarrow (arrow from $g(x)$ to $f(x)$)
 anti: \leftarrow (arrow from $f(x)$ to $g(x)$)

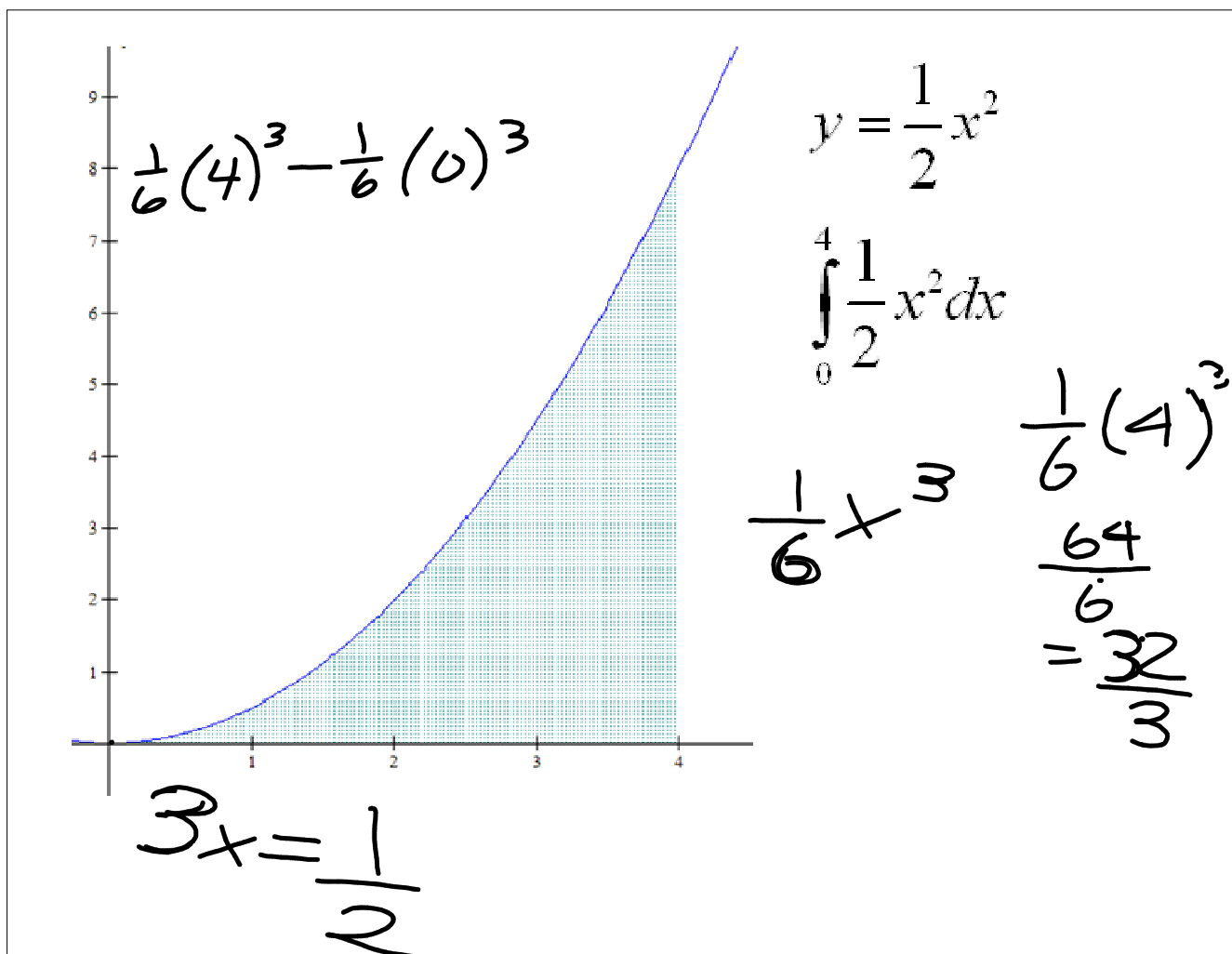


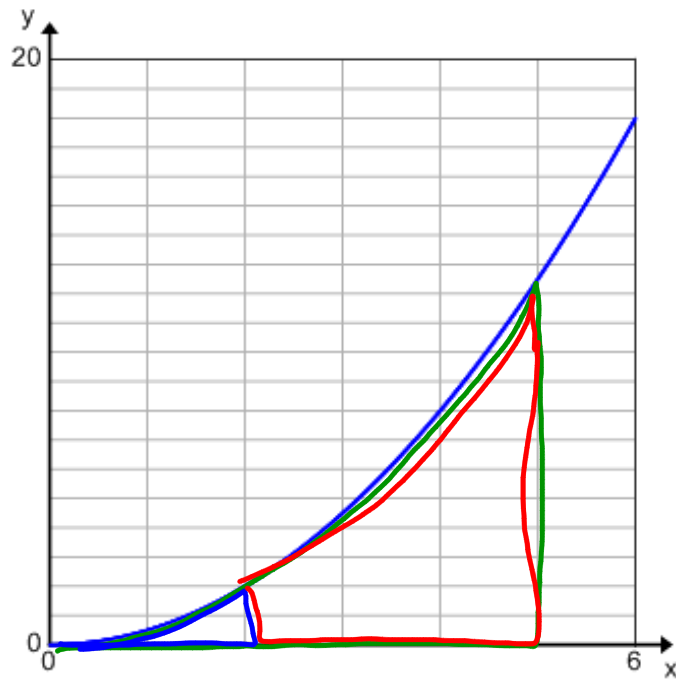
$$f(t) = 2t$$



$$g(x) = x^2$$

g(x) is the antiderivative of f(t)





$$y = \frac{1}{2}x^2$$

$$\int_2^5 \frac{1}{2}x^2 dx$$

$$\frac{1}{6}x^3$$

$$\frac{1}{6}(5)^3 = 125/6$$

$$\frac{1}{6}(2)^3 = 8/6$$

$$\frac{117}{6} = \boxed{\frac{39}{2}}$$

The FUNdamental Theorem of Calculus Part 2

If $f(x)$ is continuous at every point in $[a,b]$ and if $F(x)$ is the antiderivative of $f(x)$ on $[a,b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Ex1. Evaluate. Check your answer on the graphing calculator

$$1.) \int_{-2}^5 (x^2 + 3x + 1) dx$$

Handwritten solution for the integral problem:

Formula: $\int_a^b f(x) dx = F(b) - F(a)$

1. $\int_{-2}^5 (x^2 + 3x + 1) dx$ $F = \frac{x^3}{3} + \frac{3x^2}{2} + x$

$\left[\frac{1}{3}x^3 + \frac{3}{2}x^2 + x \right]_{-2}^5$

$F(5) - F(-2)$

Calculation:

$$= \left(\frac{125}{3} + \frac{75}{2} + 5 \right) + \left(\frac{8}{3} + 4 \right)$$

$$= \frac{133}{3} + \frac{75}{2} + 1$$

$$\frac{266}{6} + \frac{225}{6} + \frac{6}{6} = \frac{497}{6}$$

$$2.) \int_1^4 (e^{2x}) dx \quad \frac{1}{2} e^{2x} \Big|_1^4$$
$$\frac{1}{2} e^{(2 \cdot 4)} - \frac{1}{2} e^{(2 \cdot 1)}$$
$$\boxed{\frac{1}{2} e^8 - \frac{1}{2} e^2}$$

$$3.) \int_0^{\pi} \sin x dx$$

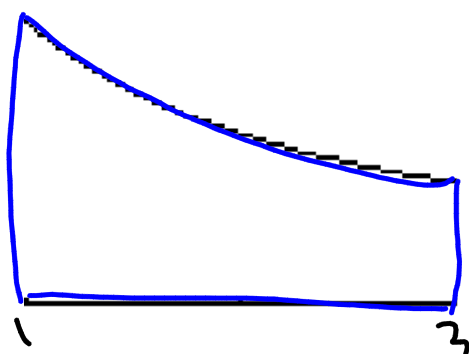
$$F(x) = -\cos x \Big|_0^{\pi}$$
$$-\cos^{-1} \pi - (-\cos 0)$$
$$1 + 1 = 2$$

$$\begin{aligned} 4.) \int_{-1}^3 \frac{dx}{x+5} &= \int_{-1}^3 \frac{1}{x+5} dx \\ &= \ln|x+5| \Big|_{-1}^3 \\ &= \ln|3+5| - \ln|-1+5| \\ &= \ln|8| - \ln|4| \\ &= \ln 8 - \ln 4 = \ln\left(\frac{8}{4}\right) = \boxed{\ln 2} \end{aligned}$$

Ex2. Find the area between the curve

$$f(x) = \frac{1}{2x+1} \text{ and the } x\text{-axis}$$

bounded by $1 \leq x \leq 3$



$$\int_1^3 \frac{1}{2x+1} dx$$

$$\left. \frac{1}{2} \ln |2x+1| \right|_1^3$$

$$\frac{\frac{1}{2} \ln 7 - \frac{1}{2} \ln 3}{\frac{1}{2} \ln \frac{7}{3}}$$

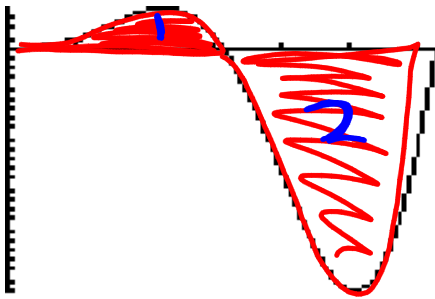
$$\ln \left(\frac{7}{3} \right)^{1/2}$$

$$\ln \sqrt{\frac{7}{3}}$$

Ex3. Find the area between the curve

$$f(x) = x^2 \sin x \text{ and the } x\text{-axis}$$

bounded by $0 \leq x \leq 2\pi$



(1) $\int_0^{\pi} x^2 \sin x \, dx = 5.8696$

(2) $\int_{\pi}^{2\pi} x^2 \sin x \, dx = -45.3480$

$45.384 + 5.8696$
 ≈ 51.218

OR

$$\int_0^{2\pi} |x^2 \sin x| \, dx$$

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Plot1 Plot2 Plot3
\Y1=X^2 sin(X)
\Y2=|Y1|
\Y3=
\Y4=
\Y5=
\Y6=
    
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Homework

pg 302 # 27, 29, 30, 32-35, 38,
39, 42, 43, 45-50